Our Knowledge by Acquaintance with the Natural Numbers

1. Introduction

Consider the following case. Emily is taking a maths exam and is asked the following question: what is 57 x 12? Emily writes in the expression '57(12)' in a mixed base 10 and base 12 notation as her answer. But Emily gets a zero for this answer, for this is not the answer the examiner was looking for. Rather the examiner was looking for the expression '684'. Indeed, one can imagine the examiner not only not giving Emily any points, but that Emily's answer elicits the marker's ire. For Emily's answer reveals that though she is rather clever, she is nevertheless woefully unprepared.

Why is this? Why is the expression '57(12)' not an acceptable answer, but the expression '684' is an acceptable answer? After all, both expressions are singular terms that denote the same number and that number is indeed the product of 57 and 12. That is to say, the two terms are coextensive, and since they are coextensive, they are informationally equivalent, at least in the minimal sense that they rule out the same possibilities. Moreover, there seems to be nothing in the question 'what is 57 x 12?' that would make the expression that completes it referentially opaque, to use a phrase of Quine's. That is, there is no obvious attitude verb (propositional or otherwise), nor a modal operator, nor anything else to indicate that this sentence is a non-extensional context. Indeed, '57(12)' is coextensive with '684'. How then, can the examiner be justified in such discrimination against Emily's answer?

I aim to do two things in this paper. First, I aim to show that this is a puzzle about reference and the epistemic role referential devices can play for cognitive creatures like us. Put another way, I aim to show that, though this is partly a puzzle in the philosophy of maths, it has important consequences for issues in the philosophy of language, mind, and epistemology. Second, I aim to show that the solution to this puzzle is a proper understanding of knowledge by acquaintance. Knowledge by acquaintance is undergoing a renaissance of study at the moment[1]. Despite this renaissance, most, if not all, research on knowledge by acquaintance is focused on perceptual knowledge and self-knowledge[2]. As important as these areas are to any account of knowledge by acquaintance, we prevent ourselves from developing an adequate account of knowledge by acquaintance by focusing only on them. I aim to show that a more robust theory of knowledge by acquaintance can be developed once we consider other kinds of knowledge by acquaintance beyond perception and self-knowledge. In particular, I aim to show that we have knowledge by acquaintance with the natural numbers.

The plan to achieve these aims is as follows. In section two, I lay out the puzzle in more detail. I show how, despite some similarities, it is not a case of Frege's puzzle but a puzzle about *knowing which*. In section three, I give my solution to the puzzle in terms of knowledge by acquaintance. I argue that knowledge by acquaintance is a form of *knowing*

[1] See Knowles and Raleigh (2019) for a recent collection of essays on acquaintance.

[2] See Duncan (2021) for a good taxonomical overview of different acquaintance theorists.

which[3]. I then show how numerals are referential devices that enable knowledge by acquaintance with abstract objects like numbers. To do this, I draw on Kaplan's (1968) theory of vivid names and Kripke's (2011) theory of revelatory senses. In section four, I conclude.

2. The Puzzle

Our puzzle is to explain how the examiner can be justified in disallowing Emily's response '57(12)' to the question 'what is 57 x 12?'. It is puzzling because the natural number that the expression '57(12)' denotes is 684, and 684 is the right answer to the question. In this section, I will do two things. First, I will argue that the puzzle is not a case of Frege's puzzle. Second I will argue that the puzzle is about *knowing which*.

Recall that Frege's puzzle is about the informativeness or cognitive significance (I use these terms interchangeably) of two co-extensive expressions that can be substituted *salva veritate.* For instance, consider the following two sentences:

(1) The author of *Hamlet* is the author of *Hamlet.*
(2) The author of *Hamlet* is William Shakespeare.

Sentence (1) is not informative or cognitively significant in at least the following sense: if someone told you (1) you would not be surprised or learn anything new. It is not cognitively significant for you because it carries no new information. It is a trivially true sentence of the form A = A. By contrast, sentence (2) is informative. Assuming you did not know Shakespeare wrote *Hamlet,* that sentence informs you of something new.

What Frege found puzzling about these kinds of sentences is that all the singular terms are co-extensive. That is, they pick out the same object. In our example, they all pick out the English writer William Shakespeare. This is puzzling because, at least on one common (Millian) understanding of names and singular terms, the only meaning is their referent. So, if we have two sentences with the same singular terms and the same form or structure, the meaning of the two sentences should be the same. But if they have the same meaning, then how can (2) be informative whereas (1) is not? This is, in a very compressed form, Frege's puzzle.

In some respects, our puzzle may look like a version of Frege's puzzle. After all, both puzzles have co-extensive terms flanking an identity sign. But they are not the same puzzle. One reason why they are not is that in our puzzle neither the examiner nor the student are lacking in information. The examiner already knows '57 x 12' denotes the same number as '57(12)', as does Emily. The example was not that Emily was just *guessing.* It was that she was refusing to answer the question in the way the question required. So, the cognitive significance or informativeness of these expressions is not what is at issue here.

To see this more clearly, I will spell out the nature of the question/answer situation Emily finds herself in. One way to understand questions is in terms of ignorance (Fiengo 2007). Asking questions can be a way to relieve oneself from ignorance. For instance, if I am ignorant of what phone number to dial to reach you, then asking you the question

[3] Evans (1982) was one of the first to argue that knowledge by acquaintance is a form of knowing which. Though my view has similarities to his, I depart from him in significant ways, as will be shown below.

'what is your phone number?' is an attempt to relieve that ignorance of mine. If you tell me truly what your number is, I am no longer ignorant (assuming that I believe you, that you are trustworthy, and so on). Thus, asking questions and receiving answers can be understood as a relief from a lack of information, a relief from ignorance.

Notice that this is not what is going on in Emily's situation. The examiner's question is not an attempt to relieve her ignorance of the product of 57 and 12. The examiner already knows what the product is. Her asking is not to relieve *that* lack of information. Rather, she wants to know whether or not the student knows which number it is and, moreover, she wants the student to demonstrate this knowledge to her appropriately. To see this, compare Emily's situation to an experience from my childhood that I think will be familiar to many readers. When I was a child, before I was allowed to leave the house, my mother would quiz me as to what our home phone number was (this was before mobile phones were widely available). She obviously didn't do this because she didn't know the number herself. Rather, she wanted to make sure I knew it. For this reason, an answer such as, 'Yes Mum, I know it' would not satisfy her. What she wanted was the number recited so that my knowledge of it was manifested in the answer I gave. Any answer that did not manifest this knowledge was disallowed and I was not allowed out. Emily finds herself in a similar situation. The examiner requires of her an answer that manifests her knowledge of the product. What this highlights is that there are epistemic constraints on any answer given in this type of question and answer situation. The question is not about ruling out possible answers to the question. My mother is not trying to rule out one phone number from another. The examiner is not trying to rule out 685 or 683 as answers. *Those* questions are already settled for the questioner. My mother and the examiner already know the answers to their respective questions. What they want are answers that manifest mine or Emily's knowledge. What this means is that the answers in these situations have epistemic constraints on them. It is not appropriate to merely provide the questioner with a means of knowing the answer. One must supply the answer in a way that manifests knowledge. Thus, one key difference between our puzzle and Frege's is that Frege's puzzle is about the informativeness of sentences that are used (or can be used) to relieve ignorance, while our puzzle is about using expressions that manifest a certain kind of knowledge.

What kind of knowledge does Emily need to manifest? She needs to manifest knowledge of *which* number it is. What the examiner wants to know is if Emily knows which number is the product of 57 and 12. This requires using expressions that manifest this fact. What the examiner wants to know is whether or not the student is capable of finding the product. Indeed, not only does she want to know whether or not the student has this ability, but she wants the student to exercise this ability and demonstrate that she has exercised this ability in the given case. Such an exercise would *manifest* knowledge of which number is the answer. As it stands, Emily has failed to demonstrate that she knows which number is the product of 57 and 12. She has failed to demonstrate this because the expression '57(12)' does not tell you which number it is, namely 684 (as opposed to 685 say)[4].

One response to this puzzle which I think is unsatisfactory but is sometimes mentioned is that the 'knowing which' locution is merely an artifice of the exam context. That is to say, what the student really needs to show is the knowledge *that* some numeral in the same base is the right answer. Asking Emily *which* number is the product helps Emily understand the task, but really she doesn't need to know which number it is, she just needs to know that some numeral is the correct answer. A certain numeral notation is required by the examiner because that is the real pedagogical task. The student needs to demonstrate proficiency with this numeral system. There is perhaps, this response says, an implicature that the student answer in Arabic numerals or decimal notation and not Roman numerals or something else. Much like if one takes a Spanish language test and answers in another language, one would receive no marks. Thus, what is really going on is that the student needs to demonstrate knowledge *that* some numeral is the right answer *in the same base.*

This response is undercut by reflection on the nature of computation. It is no doubt true that part of Emily's task is to answer using Arabic numerals, and that part of what she demonstrates when she answers correctly is the knowledge *that* a certain numeral in that notation is the correct answer. But what it is to compute a function is to *find* the number in the number line. Boolos, Burgess, and Jeffrey (2007) make this point at extended length in their chapter on Turing computability. They acknowledge that a computation can be done in many different numeral systems and that some systems might be easier to perform such computations than others. But they point out that what a computation is doing, that is, what it is for something to be *computable*, is to be able to specify *which* number it is. To make the point vivid, they say the following:

> At each stage of the computation, the computer (that is, the human or mechanical agent doing the computation), is *scanning* some one square of the tape…If you like, think of the machine quite crudely as a box on wheels which, at any stage of the computation is over some square of the tape. The tape is like a railroad track; the ties mark the boundary of the squares; and the machine is like a very short car, capable of moving along the track in either direction (2007, 25)

The machine then scans the number line until it finds the number and halts. This metaphor is supposed to make vivid what it is to compute a function. It is to *find* the number in the number line. Finding the number is determining which number it is. Knowing that *that* numeral is the right symbol is derivative on the machine being in the right position, on knowing which position in the number line satisfies the function. Thus, what it is to

[4] A similar issue is discussed by Carnap (1947). He points out that certain expressions are epistemically privileged referential devices because they 'give the extension' and thus tell us *which* object it is. The problem with Carnap's proposal is that it does not give us an account of what it is for an expression to 'give' its extension, nor why names give their referents but descriptions do not. For reasons of space, I cannot examine Carnap's interesting proposal here.

compute a function, what it is to find the product of 57 x 12 is to know which number on the number line is the correct one[5].

In short, our puzzle is about manifesting knowledge of which number is the product. Superficially, this may seem like a case of Frege's puzzle. But the informativeness of certain sentences over others is not what is at issue here. What is at issue is Emily's ability to find which number it is, and to manifest that knowledge appropriately.

3. The Solution

We now see that Emily must manifest her knowledge of which number is the product and that '684' is the numeral that is to be used to do this. But why? What is so special about '684'? The answer, I want to suggest, is that it *enables* one to have knowledge by acquaintance with the referent. In this section, I spell out that answer in more detail. I draw on Kaplan's (1968) account of vivid names and Kripke's (2011) account of revelatory senses. But first, let me say a bit more about the nature of knowledge by acquaintance.

There are at least two key features of knowledge by acquaintance. First, knowledge by acquaintance is a presentational relation between subject and object. As Russell puts it:

> The relation of subject and object which I call acquaintance is simply the converse of the relation of object and subject which constitutes presentation. That is, to say that *S* has acquaintance with *O* is essentially the same thing as to say that *O* is presented to *S*. (1910-11, 108)

Second, knowledge by acquaintance is logically independent of knowledge of truths or propositions. That is, knowledge by acquaintance is not reducible to knowledge of propositions. Russell puts it like this:

> Knowledge by acquaintance is essentially simpler than any knowledge of truths, and logically independent of knowledge of truths, though it would be rash to assume that human beings ever, in fact, have acquaintance with things without at the same time knowing some truth about them. (1912, 46)

This does not mean that when we are acquainted with something we have no propositional knowledge of that thing. Indeed, Russell admits that it is likely, as a contingent fact about human cognition, that when we are acquainted with something we *always* have propositional knowledge about that thing. Nevertheless, knowledge by acquaintance is logically independent of any and all such propositions about the item in question.

How is knowledge by acquaintance related to our puzzle? Evans (1982) argued that we should understand knowledge by acquaintance as a form of *knowing which*. He calls this idea Russell's Principle and says the following:

> Russell held the view that in order to be thinking about an object or to make a judgement about an object one must *know which* object is in question—one must *know which* object it is that one is thinking about. (65, emphasis in original)

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[5] See also Kripke 2011b for a good discussion of computability and the 'knowing which' locution.

And in the next chapter, he says:

> In order to make Russell's Principle a substantial Principle, I shall suppose that the knowledge which it requires might be called *discriminating knowledge:* the subject must have a capacity to distinguish the object of his judgement from all other things. (89, emphasis in original)

It is not entirely clear what Evans means by the phrase 'all other things'. It would be uncharitable to read the claim as saying that the subject must be able to discriminate it from everything in every context. That would be too strong a claim. However, we can be charitable to Evans here by noting that quantifiers are notoriously context-sensitive. For instance, 'I drank all the soda' would rarely be taken to mean I drank all the soda in the world. It means, rather, all the soda relative to a domain of quantification such as the soda we had in the house. Read this way, Evans's claim is more plausible. Discriminating knowledge is the capacity to discriminate the object from all other objects in the relevant domain or context. This has clear applicability to our puzzle. The relevant context would be the natural numbers and discriminating knowledge would be discriminating one number from the others.

Notice that the way the context shifts will matter in at least the following couple of ways. One way is if there is some sort of obstruction to the subject—they are drunk, or on LSD, or a scientist is manipulating their brain in various ways, or, less dramatically, they are just exhausted—such that the subject loses their ability to discriminate in that situation. Think, for instance, of driving home exhausted and not being able to always see the traffic signs. Yet another way the context shifts is when the environment surrounding the object is somehow unfavourable. For instance, you may be able to be acquainted with Lizzy the lizard when she is on the white wall in your living room, but when she gets outside, she blends into the soil so well that you cannot spot her anymore.

A similar situation happens with numbers. We will most likely be able to know which number is presented by decimal notation, but perhaps not if they were in Roman numerals or in a notation with a different base. This is because certain systems of numerals are what Kripke calls canonical forms of notation (2011c). Each numeral in a canonical notation has an immediately revelatory sense.

> A sense is revelatory of its referent if one can figure out from the sense alone what the referent is. Both 'nine' and perhaps even 'the square of three' do have revelatory senses. Given that one can understand them, one can tell what the referent is. The same holds for 'George W. Bush' and almost for 'the father of G. W. Bush's (biological) children' though in the latter case, strictly speaking, one has to know that George W. Bush is male and has children. (2011a, 260).

Moreover, Kripke makes a further distinction between types of revelatory senses:

> A sense is *immediately* revelatory if no calculation is required to figure out its referent. If *f* is a non-computable mathematical function than the sense of 'f(n)'

> might be revelatory in the weak sense that no empirical information is required to find the referent, though perhaps a mathematical argument is needed to do so. More important, even a computable function may not yield an immediately revelatory sense. For example, 'the square of three' does not have an immediately revelatory sense, since a computation, in this case a very easy one, is required to obtain its value…'nine' however is immediately revelatory (2011a, 261).

A sense is non-revelatory where the referent is not known though the expression is understood. For instance, 'the first human born in 2050' is, as of now, an expression with a non-revelatory sense. As we have seen, 'the square of 3' is revelatory but not immediately so. It is revelatory because no empirical information is needed to determine the referent (unlike 'the first human born in 2050'). Thus, one can figure out the referent from the sense of the term alone, though it may take some calculation. Finally, our numerals, such as '9' are immediately revelatory because not only do they not require any empirical information, but they require no computation either. '9' is immediately revelatory because we know which number that is, simply by being given the numeral. Thus, referring expressions are ranked in terms of their epistemic properties, particularly the difficulty of knowing which number it is.

According to Kripke (forthcoming) numerical terms with immediately revelatory senses are 'buck-stoppers' in the following sense: once a buck-stopper has been given one cannot ask the further question, yes but *which* number is that?[6] Once given a numerical term, such as '9', there is no further question, yes but which number is that? Returning to our initial puzzle, Emily lost points because the expression she used was not a buck-stopper, and it was not a buck-stopper because it was not immediately revelatory of 684, although it has the same referent as '684'. The examiner can rightfully ask Emily, 'yes but which number is that?' and some further calculation may be required. The distinction matters beyond just the classroom. Imagine, for instance, the (now former) health secretary Mathew Hancock being interviewed about how many more deaths from CO-VID 19 there are today as compared to yesterday and him answering in some mixed base 10 base 7 notation. The interviewer, as well as the public, would be rightly angered at this cheap trick to evade the question (though it would perhaps be naive to be surprised). One would rightly press him, 'yes but *which* number is *that?'* A fully revelatory designator does not leave it open how many that is. It settles the question of *which* number it is.

The main problem with Kripke's account is that we are not told what revelation is or what it means to know which. His account is not devoid of psychological elements; he clearly ties his notion of sense to the need to perform a calculation. But it is unsatisfactory not least because it fails to tell us why not needing to perform a calculation is so important. We are not told what revelation is other than a lack of calculation. How does this work? Why does this not require calculation? Moreover, knows which is left more or less unexplained. So while Kripke gives us the start of an answer, it nevertheless fails to capture what is essential to the solution. We might put it like this: it is not that this view is wrong, but that it is only a partial solution.

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[6] For a good discussion of Kripke's forthcoming work, see Steiner (2011).

We can supplement Kripke's account with Kaplan's (1968) theory of vivid names. Vivid names are names that let the agent know which object its referent is. What is interesting about vivid names is how Kaplan spells out vividness in terms of mental representation.

'Vividness' is a term of art explained in the context of the different ways in which photographs (and other representational products) relate to their subject.

> The notion of a vivid name is intended to go to the purely internal aspects of individuation. Consider typical cases in which we would be likely to say that Ralph knows x or is acquainted with x. Then look only at the conglomeration of images, names, and partial descriptions which Ralph employs to bring x before his mind. Such a conglomeration, when suitably arranged and regimented, is what I call a vivid name. As with pictures, there are degrees of vividness and the whole notion is to some degree relative to special interests. The crucial feature of this notion is that it depends only on Ralph's current mental state, and ignores all links whether by resemblance or genesis with the actual world. If the name is such that on the assumption that there exists some individual x whom it both denotes and resembles we should say that Ralph knows x or is acquainted with x, then the name is vivid. (Kaplan, 1968, 199)

Vivid names then are names whose use involves a conglomeration of mental representations that in some way mirror or reflect the thing they represent, if they represent anything at all. What is useful about this proposal is the way Kaplan appeals to mental representation. It is because of a certain way that things show up in the subject's mind that she has a certain epistemic relation to the thing. This connects with Russell's idea that knowledge by acquaintance is being *presented* with the object. The central feature is that the object shows up in a particularly revealing or vivid way for the subject.

However, there are two features of Kaplan's vivid names that we should reject if we are to use this to solve our puzzle. First, we should reject the claim that vividness is a continuous notion. Since the vividness of a name is explained by the vividness of a representation like a photograph, vividness is continuous, like turning the resolution up or down on your screened device. Such an idea is, if not wholly inapplicable, nevertheless awkward and forced when dealing with expressions designating numbers. Consider the following four designators for the number 684.

1) 'Martha's favourite number'

2) '57(12)'

3) '685-1'

4) '684'

Each one is in some sense easier to grasp than the previous one. But should this ease be explained in terms of a continuous 'turning up' of the resolution? Such concerns seem inapt at best. The reason why is that these notions are discrete, not continuous.

Second, Kaplan puts too much weight on the current mental state. In this way, the account is too psychological. It is unconvincing that so much should depend on internal individuation aspects. Surely what matters most is not just how clear the representation is in someone's mind but, rather how much the representation accurately reflects what is represented. This is a feature of the semantics of vivid names that Kaplan is missing. It is this accuracy, this mirroring of the representation and the represented gives us special epistemic access[7].

Nevertheless, if we blend (i) the idea from Kaplan that what is essential is the way the mental representation presents the object to the subject, and (ii) the idea from Kripke the idea that certain expressions in canonical notation are revelatory of their referents, then we get a picture that can answer our puzzle. Numerals in canonical notation reveal their referents to us by a mental representation that is particularly vivid or revealing. In this way, the numerals *enable* us to be acquainted with the referent. This is a form of knowing which number it is. Moreover, it is because Emily failed to use such a numeral in a canonical notation that she failed to manifest her knowledge by acquaintance of that number. She may have had such acquaintance, but without using the proper numeral, she does not convey that knowledge of hers to the examiner. Thus she loses points on the exam.

4. Conclusion

We began with a puzzle about why certain expressions would answer a maths question and others would not. We saw that certain expressions are accepted answers because of the type of knowledge they manifest. The question then became how expressions could serve to manifest knowledge in such a way. I have tried to answer this in my account by appealing to two theses, one semantic and one psychological. The semantic thesis is that certain names for the numerals are in a canonical notation because they reflect the ordinal properties of numbers and thus allow epistemic access to their referents. The psychological thesis is that such knowledge is a particular mental state of knowledge by acquaintance, which is understood as a discriminatory capacity to know which object is which. These two theses, the semantic and psychological, are a blend of proposals we saw in the work of Kaplan and Kripke and applied to knowledge by acquaintance.

A final word about what this view does not entail. First, this view is silent on the metaphysics of numbers. I have spoken throughout as if numbers were objects in a platonic sense. It should be obvious how my view is consistent with that. But one need not be a Platonist to accept this epistemology. For instance, a nominalist may think there are no numbers, so arithmetical statements, if true, are not true by reference to a domain of numbers. If one is taken by such nominalism, one can easily replace what is known by the subject not as a number but as a numeral in a canonical notation. Thus, what Emily knows is which numeral serves as the product in arithmetical statements like 57 x 12 =?. In some ways, this metaphysics is easier for my account as there is nothing 'behind' or 'beyond' the

[7] There is much more to say about Kaplan's theory, especially how vivid names relate to what he calls 'standard names'. However, for reasons of space, I cannot address them fully here. For a good critical discussion see Ackerman (1978)

symbol, it is all signifier and signified wrapped into one. Finally, structuralists, of both eliminativist and *ante rem* varieties can accept my account. If what numbers are are just positions in a structure, then what is known is which position in the structure, however one understands 'structure'.

Finally, this account does not say why, *for us*, the Arabic numerals provide knowledge by acquaintance, as opposed to say the Roman numerals. There may be many canonical notations that could have been the ones that provide us with knowledge by acquaintance with the numbers. If we were a different species perhaps a base 7 would provide us with such knowledge. Perhaps there is an evolutionary reason why *this* canonical notation works for us. This is an interesting question, and there may be ways to answer it that involve either evolutionary psychology or perhaps anthropology[8]. However, what I hope to have shown is that there is indeed such thing as canonical notation that provide us with a special epistemic access to the objects. That is to say, I hope to have shown how it is possible that *we* can have knowledge by acquaintance with the natural numbers. It is *our* knowledge by acquaintance with the natural numbers that has been examined here.

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[8] See Chrisomalis (2010) for an interesting anthropological study of the history of numerical notation.

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